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This research in mathematical physics concerned the fundamental structure of two-dimensional quantum field theory, dealt with the interactions of a particle with a field (the "magnetic polaron"), with application to the production of spin-polarized particles from thin foils. Finally, new work on an exactly soluble antiferromagnet: an exact quantum mechanical ground state and two phases identified one with long range order and the other, a "quantum liquid." Other work in mathematical physics included renormalization group studies of certain magnetic systems.

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"Effects of Boundaries and Collisions with Theory of Solitons,"

and

"Quantum Solitons and Strange Solitons in Many-Body Problem"

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Abstract

This research in mathematical physics had several consequences. Some of the work concerned the theory of electrons and the fundamental structure of two-dimensional quantum field theory. It also dealt with the interactions of a particle with a field (the "magnetic polaron"), with possible application to the production of spin-polarized particles from thin foils. Finally, much new work was developed on an exactly soluble model antiferromagnet: an exact quantum mechanical ground state was obtained and two possible phases identified one being the usual (with long range order) and the other, a model "quantum liquid." Other work in mathematical physics was initiated, e.g., renormalization group studies of certain magnetic systems.

Statement of Work Done and Personnel

A large number of research projects were launched in this, the first year of our Air Force sponsored research in mathematical physics at the University of Utah.

The list of personnel in our research group, directly or indirectly in 1980-81 includes the following:

P.I.	D.C. Mattis, Ph.D.
	B. Sutherland, Ph.D.
	B.S. Shastry, Ph.D.
	John Bruno, Ph.D.
Consultants	D. Campbell, Ph.D.
	R. Raghavan, Ph.D.
	R. Schilling, Ph.D.
Student	M. Farid, B.S.

Much of the work has been written down and submitted for publication. Only a small fraction has actually appeared. But among these publications was 300 page book:

"The Theory of Magnetism, I: Statics and Dynamics" by D.C. Mattis (Springer, 1981).

An encyclopedia article:

"Many-Body Theory" by D.C. Mattis, pp. 567-572 in Encyclopedia of Physics, Lerner and Trigg, Eds., Addison-Wesley, 1981.

Some conference proceedings (16th International Conference LT-16):

"Exact Green Functions in Magnetic Semiconductors," B.S. Shastry and D.C. Mattis, p. 73, Vol. 107, Physica B + C.

"Quantum Percolation in Dilute Lattice of Various Dimensionality," D.C. Mattis and R. Raghavan, ibid, p. 671.

"Exact Ground State of a Quantum-Mechanical Antiferromagnet," B.S. Shastry and B. Sutherland, ibid, p. 1069.

and regular publications:

"Eigenfunction Localization in Dilute Lattice of Various Dimensionalities," R. Raghavan and D.C. Mattis, Phys. Rev. B23, 4791 (1981).

"Strange Solution to Field Theory in One Spatial Dimension," D. Mattis and B. Sutherland, J. Math. Phys. (Summer 1981).

"Ambiguities with Relativistic Delta-Function Potential," B. Sutherland and D.C. Mattis, Phys. Rev. A (in press).

"Theory of the Magnetic Polaron," B. Shastry and D.C. Mattis, Phys. Rev. B (in press).

"Failure of Renormalization Group Method in Semiclassical Limit," D.C. Mattis and R. Schilling, J. Phys. (Letters) (in press).

We now list a number of papers in progress, together with explanatory abstract.

Phase Transition in the Two-Dimensional Frustrated x-y Model: Scaling Equations, B. S. Shastry. We establish scaling equations for the two-dimensional x-y model with weak frustration using the electrostatic analogy of Kosterlitz and Thouless. In the limit of small disorder we find a shift in the critical temperature

$$T_c(x) - T_c(0) = -\frac{Jx}{k_B}\left[2\pi^2 - \frac{\pi}{2\ \ell n\ 2x}\right]$$

and the exponents are unchanged from the pure values.

Phase Transition in Diluted Two Dimensional X-Y Model, B.S. Shastry and J. Bruno. In recent letter José has used the replica technique to investigate a randomly diluted ferromagnetic two dimensional (2-D) X-Y model described by the Hamiltonian $H = -\sum_{<ij>} J_{ij}\cos(\theta_i - \theta_j)$ where the J_{ij}'s are random variables (> 0) and <ij> is a sum over nearest neighbor pairs. He concludes that the model undergoes a Kosterlitz-Thousless (K-T) transition in a manner analogous to the pure case, and finds that for a certain range of the disorder parameter x, the transition temperature $T_c(x)$ exceeds the pure value $T_c(0)$. We find this result surprising. Defining $T_c(x)$ as the highest temperature below which the susceptibility $\chi(x)$ diverges, it is easy to establish an upper bound on $T_c(x)$ using the inequality of Griffiths as generalized by Ginibre for the ferromagnetic diluted X-Y model. Consider the set $\{J_{ij}\}$ of random bonds and the pure set $\{J_{ij}{}^{o}\}$ where $0 \leq J_{ij} \leq J_{ij}{}^{o}$. The inequality alluded to above implies $\langle\vec{S}\cdot\vec{S}_j\rangle\{J\} \leq \langle\vec{S}_i\cdot\vec{S}_j\rangle\{J^o\}$ and summing over i,j and using the fluctuation dissipation theorem, we find $\chi(x) \leq \chi(0)$: Thus $T_c(x) \leq T_c(0)$.

In view of the above we believe that this problem needs reappraisal. We have considered this problem without the use of the replica technique and have found it possible to perform the standard Villain approximation followed by a dual tranformation for a given set of quenched (random) bonds J_{ij}. We find the vortex partition function in the form

$$Z_V = \sum_{m_i=0,\pm1\ldots} \exp[2\pi^2/k_Bt) \sum_{\langle ij\rangle} m_i m_j G_{ij}]$$

where G_{ij} is the inverse of the matrix

$$H_{ij} = \sum_{\ell} (1/\bar{J}_{i,\ell})[\delta_{i,j} - \delta^{\kappa,j}\delta_{|r_{ij}|,1}]$$

and $\bar{J}_{ij}$ is the bond variable in the original lattice which is cut by the bond joining the dual lattice sites i and j.

Spin Dynamics of the Long-Ranged Ising Sping Glass, B.S. Shastry. An analytical theory of the spin dynamics of the long-ranged Ising spin glass model is proposed using the inverse of the coordination number as an expansion parameter. In equilibrium the theory reduces to that proposed by Thouless, Anderson and Palmer. Power law decays of the autocorrelation function $(1/t^{1/2})$ and remanent magnetization $(\sim 1/t^{3/2})$ are obtained for all $T < T_c$.

Excitation Spectrum of a Dimerized Next-Neighbor Antigerromagnetic Chain, B.S. Shastry and B. Sutherland. We study the excitation spectrum of an antiferromagnetic chain exhibiting both the effects of dimerization and frustration. Our method is based on an exact solution for the doubly degenerate ground state, and views the excitations as propagating defect boundaries between the two exact ground states. These excitations are analogous to "solitons"; and can bind into a second type of excitation, analogous to "breathers."

Exact Solution of a Large Class of Interacting Quantum Systems Exhibiting Ground State Singularities, B. Sutherland and B.S. Shastry. We demonstrate how to construct a large class of interacting quantum systems for which an exact solution may be found for the ground state wave function and ground state energy for some range of interaction parameters. It is shown that the ground state exhibits singularities in these cases, and in some instances the exact ground state phase diagram and critical indices are also found.

Comment on Ordering in Multicomponent Systems, B. Sutherland. This paper demonstrates that the conclusions arrived at on the possibility of multipolar ordering in the exchange-interaction model of ferromagnetism by Chen and Joseph in a paper of the same title are not justified.

Nonuniversal Equation of State for Random Uniaxial Dipolar Ferromagnets at Marginal Dimensionality, J. Bruno and C. Vause. The critical behavior of a random uniaxial dipolar ferromganet is determined in the range of reduced temperatures $t_x \ll |t| \ll 1$, where t_x is a "crossover" temperature whose origin is in a particular symmetry (at leading order only) in the scaling equations of the fluctuation theory. The (internal) magnetic susceptibility, specific heat, equation of state and spontaneous magnetization all show nonuniversal behavior with concentration dependent exponents in this region. Comparison with recent experiments on the random uniaxial dipolar ferromagnet $LiTb_{1-p}Y_pF_4$ is given.

Failure of Renormalization Group Method in Semiclassical Limit, D.C. Mattis and Rolf Schilling. We compute the ground state energy of an Heisenberg linear chain antiferromagnet for spins $s = 1/2, 1, \ldots, \infty$ by a block spin renormalization method. Although this method yields reasonably accurate results in the quantum limit $s = 1/2$, in the semiclassical limit [where significant parameter in the asymptotic expansion $e_0 = -Js^2(1 + \gamma/2s + O(1/2)^2)$ is known to be $\gamma \approx 0.7$] the RG procedure yields $\gamma \approx 0$, a highly unsatisfactory result.

Does a Charged Defect Trap anExciton, Does the Hydrogen Atom Bind a Positron? M. Farid and D.C. Mattis. We review the binding of a positron to a neutral hydrogen, or the binding of an exciton to a charge defect center. Our upper bound is compared to Armour's lower bound. One general conclusion: If the mass ratio of the mobile particles is close to 1 (in the range 2/3 - 3/2) the exciton can never be bound to a singly charged center of either sign, whether positive or negative. If the mass ratio exceeds approximately 3/2, the exciton can be bound a fixed singly charged center of the same sign as the more massive mobile particle.

The "Strange Solutions and other Quarks of Quantum Field Theory" were the subject of a special report to the Air Force, which we reproduce herewith.

SPECIAL REPORT TO

Dr. Robert N. Buchal
AFOSR

STRANGE SOLUTIONS AND OTHER QUIRKS OF QUANTUM FIELD THEORY

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6 April, 1981

I. INTRODUCTION

Quantum field theory--the fundamental study of elementary particles--and condensed matter physics presently share many of the same mathematical techniques and solutions[1]. That is, vastly different problems are found to lead, ultimately, to equations that are surprisingly similar and allow a common method of solution. "Solitons" are an example of features appearing both in QFT as well as in such mundane stuff as polyacetylene![2]

Our work[3] concerns certain ambiguities which exist in the mathematics and which can only be resolved by a correct physical interpretation. There are two principal applications.

(1) Calculations of ground state and elementary excitations of many-fermion systems, and

(2) the nature and effect of short-range interactions.

Concerning the first, we find that in typical model problems there exist two classes of solutions, the physically acceptable ones and the "strange solutions" which we have newly identified[3]. It is even possible, in a given, fairly complex problem, to concoct solutions that are partly "strange" and partly the result of acceptable manipulations; the combination being, of course, just as unacceptable.

Concerning the second application, we find that the delta function interactions suitable to condensed matter physics are of the "separable" variety, whereas those suited to QFT are the "local" variety, and that there exist in addition an infinite number of other varieties. What makes this important is that each variety leads to a different set of eigenstates. The phase shifts for the scattering, and the nature of the bound states, all depend sensitively on the precise choice of functional form, and on certain limiting processes.

The mathematical-physics literature is a mass of confusion over these issues. Many workers, solving a common model but using different cut-off procedures, will arrive at a different answer. Others will converge onto a "strange" solution. One example is the "massive Thirring model" of QFT. Almost every attempt at solution[4] has met with a disproof[5] until, recently, when a complete, explicit solution was proposed for this knotty problem: all the eigenstates and eigenvalues are in principle capable of being extracted from this solution[6]. But it is our present belief that this is in the nature of a "strange", unacceptable solution. This is not an academic point; strange solutions will not have acceptable physical properties, and will not concord with Nature where such comparisons can be made. In condensed-matter physics, there are breakthrough claims in the knotty Kondo problem[7]. Such solutions will not be valuable, and will not permit useful comparison with experiment, if they contain a partly "strange" component.

Why have these difficulties not been identified earlier? In condensed matter physics, they arise only when the kinetic energy operator is "linearized", i.e. $-d^2/dx^2$ is replaced by $\pm iv_F d/dx$, as is appropriate only for particles at the Fermi level k_F. In QFT, these problems cannot arise when the Fermi sea is properly filled, and diagrammatic perturbation theory is used to solve the problems of interacting particles. It is only the fairly modern attempts at algebraic, closed-form solutions that can give rise to ambiguities or wrong answers.

II. STRANGE SOLUTIONS

We illustrate by the simplest model of spinless fermions in one dimension, all moving at a common speed v_0. There exists no particularly interesting application of this model; it is only used to highlight the difference between acceptable and "strange" solutions. Assuming an interaction potential $V(x_i - x_j)$, the Schrodinger equation takes the form

$$\left[- i v_0 \sum_n d/dx_n + \sum_{n,m} V(x_n - x_m)\right] \psi(x_1, \ldots) = E\psi(x_1, \ldots) .$$

Because this is a first-order p.d.e. and is linear in the wave-function $\psi(x_1, \ldots)$, it is possible to solve this equation in closed form; we shall give this result below. This is, however, the "strange" solution. The reason: the differential operator above is ill-defined. In particular, it is possible to find wave-functions with sufficient "wiggles" to make $E \to -\infty$. Thus, the Hamiltonian has no ground state, <u>every</u> state we obtain is an infinitely high excited state. In mathematical terms, we are in the wrong Hilbert space. In physical terms, we have no chance of obtaining the correct normal modes of the system, which are near the ground state. All solutions of the equation above take the form:

$$\psi(x_1, \ldots) = e^{ikx_1} \prod_n \delta(x_{n+1} - x_n - r_n) , \quad E = k + \sum_{n,m} V(r_{n,m}) .$$

The r_n's are fixed parameters, indicating fixed separation on the particles. It is easy to adapt this solution to the Pauli principle but not to the uncertainty principle. If we know the position of one particle, we know them all!

The construction of a correct solution starts with Figure 1. We first fill the Fermi sea (down to k's of $-\infty$); this brings us into an acceptable Hilbert space, which has a ground state (filled F.S.), and elementary excitations (particle above F.S., hole below). One can either sum the diagrammatic perturbation theory on the potentials or solve the problem algebraically[3]. The results are: the ground state energy is modified, and the spectrum of elementary excitations is modified; the q^{th} normal mode propagates at a suitably renormalized speed $v_0(1+V_q)$, where V_q is the Fourier transform of $V(x)$.

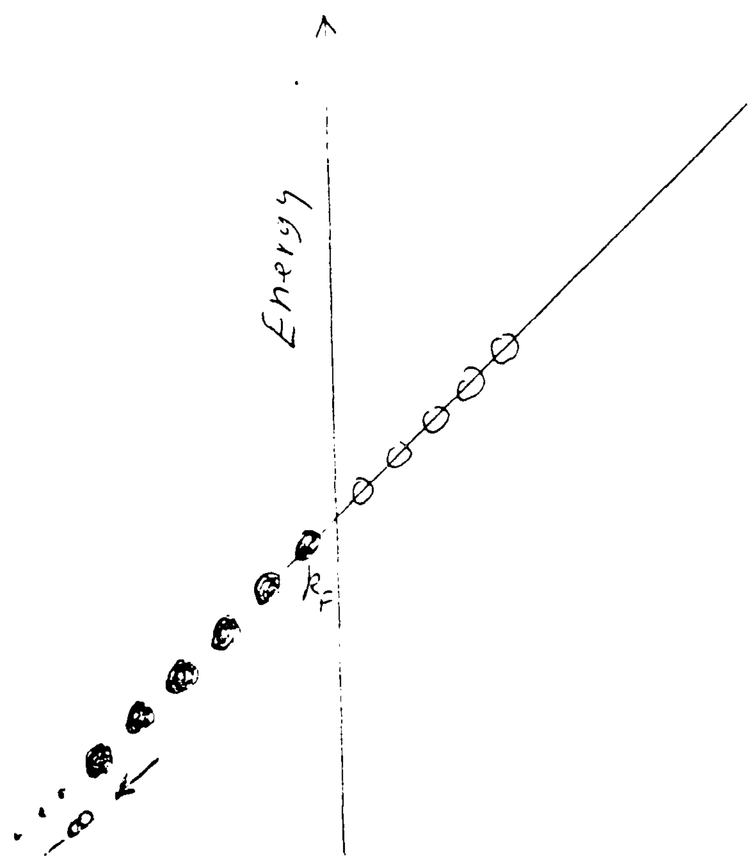

Fig. 1. The procedure of filling the F.S. eliminates the possibility of particles at large negative k being scattered, but changes the dynamic nature of the forces.

The first method is wrong, because the problem makes no sense for a finite number of particles. If we tried to modify it to make sense--e.g. by introducing a cutoff at $-k_c$, the Schrodinger's equation is properly no longer a first-order p.d.e., and we can no longer solve it conveniently. On the other hand, the procedure first discovered by Fermi and Dirac, of filling the Fermi sea, leads to a proper solution without the use of any cut-offs. Again we can speak of a finite number of particles--but they are holes below the FS and electrons above.

In condensed-matter theory applications, the linearization of the kinetic energy is not a consequence of relativity, but is a mere mathematical simplification that the theorist makes to render the many-body problem more tractable. It is then especially important that non-physical processes, occurring at large negative energies, not be allowed to take place.

A peculiar symptom of "strange solution" is that the interactions can be "gauged away". That is, if we have two colliding particles as in

$$[- id/dx + id/dy + V(x-y)]\psi(x,y) = E\psi(x,y)$$

we can write ψ in the form $[\exp - \{\frac{1}{2} i \int^{x-y} dx'V(x')\}]\phi(x,y)$, we find that ϕ satisfies the equation:

$$[- id/dx + id/dy]\phi(x,y) = E\phi(x,y)$$

with the same E as above. Thus, the interaction has no tangible, dynamic effect! The correct version of the above cannot even be formulated for two particles. If we fill the F.S. first, we obtain quite a different problem, and an entirely different set of solutions. If we do not, but merely modify the kinetic energy as in

$$[- d^2/dx^2 - d^2/dy^2 + V(x-y)]\psi = E\psi(x,y)$$

we cannot then "gauge away" the interaction. Either way, we cannot reconcile the "strange solution".

It seems as though the hallmark of "strange solutions" is the "gauging away" of difficult interaction terms. This can only be done for linearized kinetic energies, when the F.S. is not filled correctly. We are examining the published "breakthrough" solutions[6,7] for evidence of this subtle, but damaging, error.

III. TWO DELTA FUNCTIONS

We illustrate this topic by a simple example. First the well-known theorem:

$$\delta(x)\delta(x-y) = \delta(x)\delta(y) \quad . \tag{1}$$

Actaully, the l.h.s. of this equation represents a local potential V(x) in the delta-function limit, where as the r.h.s. is a separable potential, also in the delta-function limit. In problems involving second-order differential operators, the two are, indeed, identical. This is because the wavefunctions are constrained to be continuous. In problems, such as those of QFT, where the differential operators are first-order, the wave functions become discontinuous at the delta-function singularity. The two forms in Eq.(1) then yield <u>different</u> results. They are sketched in Fig. 2.

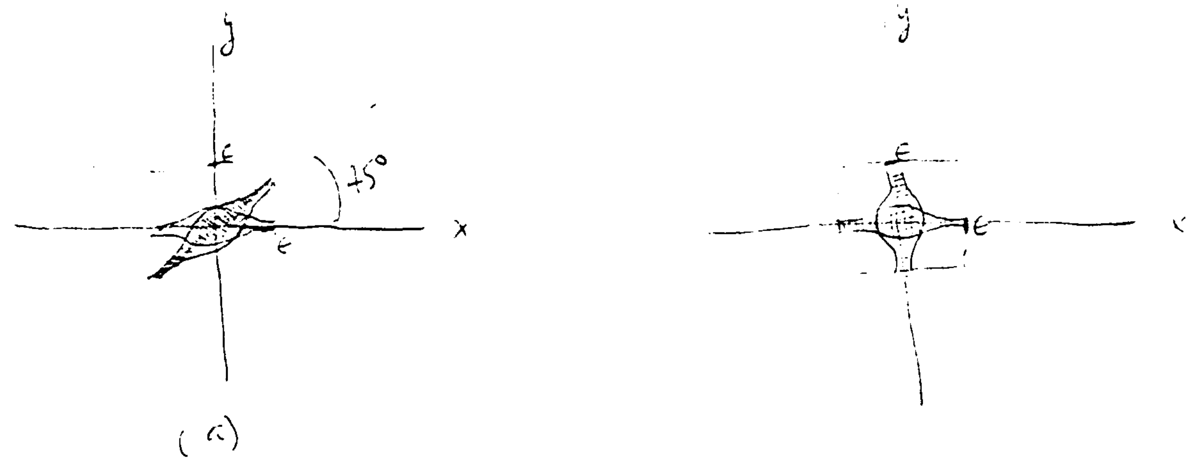

Fig. 2. (a) local (b) nonlocal. Limit $\varepsilon \rightarrow 0$ must be taken. An ∞ number of other shapes are possible, each with different results.

(a) Local. We solve

$$- i d/dx\ \phi(x) + g\delta(x) \int dy\ \delta(x-y)\phi(y) = E\phi(x)$$

and obtain:

$$\phi_{\ell}(x) = e^{-i\theta(x) + iEx}$$

where $\theta(x) = -\frac{1}{2} g$ for $x < 0$ and $+\frac{1}{2} g$ for $x > 0$. It may be verified by direct substitution.

(b) Non-local. We use the separable form, r.h.s. of (1).

$$- i d/dx\ \phi(x) + g\delta(x) \int dy\ \delta(y)\phi(y) = E\phi(x) .$$

This has the solution, also verifiable by direct substitution,

$$\phi_{n.\ell.}(x) = e^{iEx}(1-i\theta(x)) \quad (x \neq 0), \text{ and } \phi(0) = 1$$

with $\theta(x)$ defined as above. Equivalently,

$$\phi_{n.\ell.}(x) = e^{-i\lambda(x) + iEx}$$

with $\lambda(x) = -\tan^{-1}\frac{1}{2}g$ for $x < 0$ and $+\tan^{-1}\frac{1}{2}g$ for $x > 0$.
The two versions agree only for small g. At $g = 2\pi \times$ integer, the local solution is perfectly continuous at the origin, whereas the nonlocal version remains highly discontinuous. Both have transmission coefficients of unity, and are examples of the "gauging away" of interactions that we criticized in the preceding section.

The practice of introducing cut-off into singular interactions leads to behavior intermediate between the extremes of local and separable; thus in a number of research projects, it has been possible for the investigator to obtain any answer he desires, merely by "massaging" the interaction into a form which yields a desired result . Our present concern is with finding a procedure which will yield invariant and physically admissible results for short-range forces.

FOOTNOTES

1. For examples, ITP at Santa Barbara is sponsoring a workshop on "Common Problems" this summer.

2. W. P. Su and J. R. Schrieffer, Phys. Rev. Lett. 46, 738 (1981); B. Horovitz, ibid., 742.

3. Our interest dates back to D. Mattis and E. Lieb, J. Math. Phys. 6, 304 (1965). (See also Chap. 4 in Lieb and Mattis, "Mathematical Physics in One Dimension, Academic Press, New York, 1966.) Recently: D. Mattis and B. Sutherland, "Strange Solutions", J. Math. Phys. (in press), and papers being submitted.

4. Typically, an attempt to guess at the S-matrix by F. Berezin and V. Sushko, JETP, Sov. Phys. 21, 865 (1965).

5. Shot down by P. Hählen, Nucl. Phys. B102, 67 (1976).

6. H. Bergknoff and H. Thacker, Phys. Rev. D19, 3666 (1979); and Revs. Mod. Phys. (in press).

7. N. Andrei, Phys. Rev. Lett. 45, 379 (1980); P. Wiegmann, Pisma v. JETP 31, 392 (1980); Fateev and Wiegmann, Phys. Lett. 81A, 179 (1981).

We enjoyed the consultative services of several visitors, including Drs. D. Campbell (Los Alamos) and R. Raghavan (Riverside Research Institute, New York). Papers were presented at 1st Intermountain West-Southwestern Technical Physics Conference, held at Los Alamos in March 1981. The second annual meeting is scheduled for March 1982 under the general direction of D.C. Mattis.